

Generalized λ_c -Open Set

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Abstract. In this paper, we define a new class of sets which we call generalized λ_c -closed sets, and we give some of their properties.

Keyword. λ_c -open sets, λ_c -closed sets and generalized λ_c -closed sets.

Introduction. In 1970 [1], Levine N, defined generalized closed sets in topology. In 1987, Bhattacharyya P. and Lahiri B. K, define Semi-generalized closed set in topology In 1963, Levine [2], defined semi open sets and semi continuous functions in a space X , and discussed many of its properties. Sarhad Faiq Namiq & Alias B. Khalaf [3],[4],[5],[6],[7],[8],[9]. They study a new class of semi open sets, which they call a λ -open set and λ_c -open set in topological spaces and also they define the notions of λ -interior, λ -limit point, λ -derived set. In the second section, we define the notions of λ_c -interior, λ_c -limit point and λ_c -derived set of a set and they show that some of their properties are analogous to the properties in open sets. Moreover, we give some additional properties of λ_c -closure and λ_c -interior of a set. We see Willard S., General Topology [10], to study some concepts in topological space.

3.Preliminaries

In 1970 [1], Levine N, defined generalized closed sets in topology. In 1963, Levine [2] defined semi open sets and semi continuous functions in a space X . Sarhad Faiq Namiq and Alias B.Khalaf [3],[4],[5],[6], They introduce new classes of semi open sets called λ -open and λ_c -open sets in topological spaces. They consider λ as a function defined on the family of semi-open sets of X into the power set of X and $\lambda : SO(X) \rightarrow P(X)$ is called an s-operation if $V \subseteq \lambda(V)$ for each V .

The followings depended on[3],[4],[5],[6],[7],[8].

Definition 3.1

Let (X, τ) be a topological space and $\lambda : SO(X) \rightarrow P(X)$ be an s-operation. Then a subset A of X is called a λ -open set if for each $x \in A$ there exists a semi open set U such that $x \in U$ and $\lambda(U) \subseteq A$. The complement of a λ -open set is said to be λ -closed. The family of all λ -open (resp. λ -closed) subsets of a topological space (X, τ) is denoted by $SO_\lambda(X, \tau)$ or $SO_\lambda(X)$ (resp. $SC_\lambda(X, \tau)$ or $SC_\lambda(X)$).

Definition 3.2

A λ -open subset A of a topological space (X, τ) is called λ_c -open if for each $x \in A$ there exists a closed set F such that $x \in F \subseteq A$. The complement of a λ_c -open set is said to be λ_c -closed. The family of all λ_c -open (resp. λ_c -closed) subsets of a topological space (X, τ) is denoted by $SO_{\lambda_c}(X, \tau)$ or $SO_{\lambda_c}(X)$ (resp. $SC_{\lambda_c}(X, \tau)$ or $SC_{\lambda_c}(X)$).

Remark 3.3

From the definition of s-operation, it is clear that $\lambda(X) = X$ for any s-operation λ . Through out this thesis we assume that $\lambda(\phi) = \phi$, for any s-operation λ .

Definition 3.4

A subset of topological space (X, τ) is said to be λ_c -clopen if it is both λ_c -open and λ_c -closed set. The family of λ_c -clopen sets of X , denoted by $CO_{\lambda_c}(X)$.

Definition 3.5

Let (X, τ) be a topological space and let A be a subset of X . Then:

- (1) The λ -closure of A ($\lambda Cl(A)$) is the intersection of all λ -closed sets containing A .
- (2) The λ -interior of A ($\lambda Int(A)$) is the union of all λ_c -open sets of X contained in A .

- (3) A point $x \in X$ is said to be a λ -limit point of A if every λ -open set containing x contains a point of A different from x , and the set of all λ -limit points of A is called the λ -derived set of A denoted by $\lambda d(A)$.

Proposition 3.6

Let (X, τ) be a topological space and $A \subseteq X$. For each point $x \in X$, $x \in \lambda Cl(A)$ if and only if $V \cap A \neq \emptyset$ for every $V \in SO_\lambda(X)$ such that $x \in V$.

Definition 3.7

A subset A of a topological space (X, τ) is called:

- (1) A generalized closed set (g -closed) [1], if $A \subseteq U$ and $U \in \tau$ implies that $Cl(A) \subseteq U$,
- (2) A semi-generalized closed set (sg -closed) [11], if $A \subseteq U$ and U is semi open implies that $sCl(A) \subseteq U$,
- (3) a generalized semi-closed set (gs -closed) [12], if $A \subseteq U$ and $U \in \tau$ implies that $sCl(A) \subseteq U$.

Example 3.8

Let $X = \{a, b, c\}$, and $\tau = \{\emptyset, \{a, c\}, X\}$. We define an s -operation $\lambda : SO(X) \rightarrow P(X)$ as:

$$\lambda(A) = \begin{cases} A & \text{if } A = \{a, c\} \text{ or } \emptyset \\ X & \text{Otherwise} \end{cases}.$$

Then:

$$SO(X) = \{\emptyset, \{a, c\}, X\}; SO_\lambda(X) = \{\emptyset, \{a, c\}, X\}; \text{ and } SO_{\lambda c}(X) = \{\emptyset, X\};$$

Here, it is clear that $SO_{\lambda c}(X)$ is indiscrete, but $SO_\lambda(X)$ and $SO(X)$ are not indiscrete.

Example 3.9

Let $X = \{a, b, c\}$, and $\tau = P(X)$. We define an s-operation $\lambda : SO(X) \rightarrow P(X)$ as:

$$\lambda(A) = \begin{cases} A & \text{if } A = \{b\} \text{ or } \{a, b\} \text{ or } \{a, c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

Then we can easily find the following families of sets:

$$SO(X) = P(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\};$$

$$SO_{\lambda}(X) = \{\phi, \{b\}, \{a, b\}, \{a, c\}, X\};$$

$$SO_{\lambda c}(X) = \{\phi, \{b\}, \{a, b\}, \{a, c\}, X\};$$

Here, it is clear that $SO(X)$ is discrete, but $SO_{\lambda}(X)$ and $SO_{\lambda c}(X)$ are not discrete.

Proposition 3.10

For subsets A, B of a topological space (X, τ) , the following statements are true.

$$(1) \lambda_c Cl(A) \cup \lambda_c Cl(B) \subseteq \lambda_c Cl(A \cup B).$$

$$(2) \lambda_c Cl(A \cap B) \subseteq \lambda_c Cl(A) \cap \lambda_c Cl(B).$$

Proposition 3.11

For each point $x \in X$, $x \in \lambda_c Cl(A)$ if and only if $V \cap A \neq \phi$, for every $V \in SO_{\lambda c}(X)$ such that $x \in V$.

Proposition 3.12

For any subset A of a topological space (X, τ) , The following statements are true.

$$(1) X \setminus \lambda_c Int(A) = \lambda_c Cl(X \setminus A).$$

$$(2) \lambda_c Cl(A) = X \setminus \lambda_c Int(X \setminus A).$$

$$(3) X \setminus \lambda_c Cl(A) = \lambda_c Int(X \setminus A).$$

$$(4) \lambda_c Int(A) = X \setminus \lambda_c Cl(X \setminus A).$$

Theorem 3.13

Let A, B be subsets of X . If $\lambda : SO(X) \rightarrow P(X)$ is a λ -regular s -operation Then:

$$(1) \lambda_c d(A \cup B) = \lambda_c d(A) \cup \lambda_c d(B).$$

$$(2) \lambda_c Cl(A \cup B) = \lambda_c Cl(A) \cup \lambda_c Cl(B).$$

$$\lambda_c Int(A \cap B) = \lambda_c Int(A) \cap \lambda_c Int(B).$$

4.Generalized λ_c -Open Set

Definition 4.1

A subset A of a topological space (X, τ) is said to be generalized λ_c -closed (briefly. g - λ_c -closed) if $\lambda_c Cl(A) \subseteq U$, whenever $A \subseteq U$ and U is a λ_c -open set in (X, τ) .

We say that a subset B of X is generalized λ_c -open (briefly. g - λ_c -open) if its complement $X \setminus B$ is generalized λ_c -closed in (X, τ) .

Remark 4.2

The generalized λ_c -closed set and generalized closed set (resp. semi generalized closed set, generalized semi closed set) are independent, in Example 3.8, we have $\{a, c\}$ is not generalized closed set, semi generalized closed set and generalized semi closed set, but it is generalized λ_c -closed set, and also in Example 3.9, we have $\{a, b\}$ is generalized closed set, semi generalized closed set and generalized semi closed set, but it is not generalized λ_c -closed set.

In the following proposition we show that every λ_c -closed subset of X is g - λ_c -closed.

Proposition 4.3

Every λ_c -closed set is g - λ_c -closed.

Proof. A set $A \subseteq X$ is λ_c -closed if and only if $\lambda_c Cl(A) = A$. Thus $\lambda_c Cl(A) \subseteq U$ for every $U \in SO_{\lambda_c}(X)$ containing A .

The converse of Proposition 4.3 is not true in general and now give an example of a g - λ_c -closed set which is not λ_c -closed.

Example 4.4

Let $X = \{a, b, c\}$, and $\tau = P(X)$. We define an s -operation $\lambda : SO(X) \rightarrow P(X)$ as:

$$\lambda(A) = \begin{cases} A & \text{if } A = \{a\} \text{ or } \emptyset \\ X & \text{Otherwise} \end{cases}.$$

Then, if we let $A = \{a, b\}$, and since the only λ_c -open supersets of A is X , so A is g - λ_c -closed but it is not λ_c -closed.

Proposition 4.5

The intersection of a g - λ_c -closed set and a λ_c -closed set is always g - λ_c -closed.

Proof. Let A be g - λ_c -closed and F be λ_c -closed. Assume that U is λ_c -open set such that $A \cap F \subseteq U$, set $G = X \setminus F$. Then $A \subseteq U \cup G$, since G is λ_c -open, then $U \cup G$ is λ_c -open and since A is g - λ_c -closed, then $\lambda_c Cl(A) \subseteq U \cup G$. Now by Proposition 3.10, $\lambda_c Cl(A \cap F) \subseteq \lambda_c Cl(A) \cap \lambda_c Cl(F) = \lambda_c Cl(A) \cap F \subseteq (U \cup G) \cap F = (U \cap F) \cup (G \cap F) = (U \cap F) \cup \emptyset \subseteq U$.

Note 4.6

The union of two $g-\lambda_c$ -closed sets need not be $g-\lambda_c$ -closed, as shown in the following example:

Example 4.7

Let $X = \{a, b, c\}$, and $\tau = P(X)$. We define an s-operation $\lambda : SO(X) \rightarrow P(X)$ as:

$$\lambda(A) = \begin{cases} A & \text{if } A = \{a, b\} \text{ or } \{a, c\} \text{ or } \{b, c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

Then, if $A = \{a\}$ and $B = \{b\}$. So, A and B are $g-\lambda_c$ -closed, but $A \cup B = \{a, b\}$ is not $g-\lambda_c$ -closed, since $\{a, b\}$ is λ_c -open and $\lambda_c Cl(\{a, b\}) = X$.

Theorem 4.8

If $\lambda : SO(X) \rightarrow P(X)$ is a λ -regular s-operation. Then the finite union of $g-\lambda_c$ -closed sets is always a $g-\lambda_c$ -closed set.

Proof. Let A and B be two $g-\lambda_c$ -closed sets, and let $A \cup B \subseteq U$, where U is λ_c -open. Since A and B are $g-\lambda_c$ -closed sets, therefore $\lambda_c Cl(A) \subseteq U$ and $\lambda_c Cl(B) \subseteq U$ implies $\lambda_c Cl(A) \cup \lambda_c Cl(B) \subseteq U$. But by Theorem 3.13, we have $\lambda_c Cl(A) \cup \lambda_c Cl(B) = \lambda_c Cl(A \cup B)$. Therefore $\lambda_c Cl(A \cup B) \subseteq U$. Hence we get $A \cup B$ is $g-\lambda_c$ -closed set.

Note 4.9

The intersection of two $g-\lambda_c$ -closed sets need not be $g-\lambda_c$ -closed, as it is shown in the following example:

Example 4.10

Let $X = \{a, b, c\}$, and $\tau = P(X)$. We define an s-operation $\lambda : SO(X) \rightarrow P(X)$ as:

$$\lambda(A) = \begin{cases} A & \text{if } A = \{a\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

Then the sets $A = \{a, b\}$ and $B = \{a, c\}$ are $g-\lambda_c$ -closed sets, since X is their only λ_c -open superset. But $C = \{a\} = A \cap B$ is not $g-\lambda_c$ -closed, since $C \subseteq \{a\} \in SO_{\lambda_c}(X)$ and $\lambda_c Cl(C) = X \not\subseteq \{a\}$.

Theorem 4.11

If a subset A of a topological space (X, τ) is $g-\lambda_c$ -closed and $A \subseteq B \subseteq \lambda_c Cl(A)$, then B is a $g-\lambda_c$ -closed set in X .

Proof. Let U be a λ_c -open set of X such that $B \subseteq U$. Since A is $g-\lambda_c$ -closed, we have $\lambda_c Cl(A) \subseteq U$. Now $\lambda_c Cl(B) \subseteq \lambda_c Cl(\lambda_c Cl(A)) = \lambda_c Cl(A) \subseteq U$. That is $\lambda_c Cl(B) \subseteq U$, where U is λ_c -open. Therefore B is a $g-\lambda_c$ -closed set in X .

The converse of the Theorem 4.11 need not be true as seen from the following example.

Example 4.12

Let $X = \{a, b, c\}$, with $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$. And let $\lambda: SO(X) \rightarrow P(X)$ be a λ -identity s-operation. If $A = \{a\}$ and $B = \{a, b\}$. Then A and B are $g-\lambda_c$ -closed sets in (X, τ) . But $A \subseteq B \not\subseteq \lambda_c Cl(A)$.

Theorem 4.13

Let $\lambda: SO(X) \rightarrow P(X)$ be an s-operation. Then for each $x \in X$, $\{x\}$ is λ_c -closed or $X \setminus \{x\}$ is $g-\lambda_c$ -closed in (X, τ) .

Proof. Suppose that $\{x\}$ is not λ_c -closed, then $X \setminus \{x\}$ is not λ_c -open. Let U be any λ_c -open set such that $X \setminus \{x\} \subseteq U$, then $U = X$. Therefore $\lambda_c Cl(X \setminus \{x\}) \subseteq U$. Hence $X \setminus \{x\}$ is $g-\lambda_c$ -closed.

Proposition 4.14

A subset A of a topological space (X, τ) is $g\text{-}\lambda_c$ -closed if and only if $\lambda_c Cl(\{x\}) \cap A \neq \emptyset$, holds for every $x \in \lambda_c Cl(A)$.

Proof. Let U be a λ_c -open set such that $A \subseteq U$ and let $x \in \lambda_c Cl(A)$. By assumption, there exists a $z \in \lambda_c Cl(\{x\})$ and $z \in A \subseteq U$. It follows from Theorem 3.11, that $U \cap \{x\} \neq \emptyset$, hence $x \in U$, implies $\lambda_c Cl(A) \subseteq U$. Therefore A is $g\text{-}\lambda_c$ -closed.

Conversely, suppose that $x \in \lambda_c Cl(A)$ such that $\lambda_c Cl(\{x\}) \cap A = \emptyset$. Since, $\lambda_c Cl(\{x\})$ is λ_c -closed. Therefore by Definition 3.2, $X \setminus \lambda_c Cl(\{x\})$ is λ_c -open set in X . Since $A \subseteq X \setminus \lambda_c Cl(\{x\})$ and A is $g\text{-}\lambda_c$ -closed implies that $\lambda_c Cl(A) \subseteq X \setminus \lambda_c Cl(\{x\})$ holds, and hence $x \notin \lambda_c Cl(A)$, a contradiction. Therefore $\lambda_c Cl(\{x\}) \cap A \neq \emptyset$.

Theorem 4.15

If a subset A of a topological space (X, τ) is a $g\text{-}\lambda_c$ -closed set in X , then $\lambda_c Cl(A) \setminus A$ does not contain any non empty λ_c -closed set in X .

Proof. We prove the result by contradiction. Let F be a λ_c -closed set such that $F \subseteq \lambda_c Cl(A) \setminus A$ and $F \neq \emptyset$. Then $F \subseteq X \setminus A$ which implies $A \subseteq X \setminus F$. Since A is $g\text{-}\lambda_c$ -closed and $X \setminus F$ is λ_c -open set, therefore $\lambda_c Cl(A) \subseteq X \setminus F$, that is $F \subseteq X \setminus \lambda_c Cl(A)$. Hence $F \subseteq \lambda_c Cl(A) \cap X \setminus \lambda_c Cl(A) = \emptyset$. This shows that, $F = \emptyset$ which is a contradiction. Hence $\lambda_c Cl(A) \setminus A$ does not contains any non empty λ_c -closed set in X .

Lemma 4.16

Let A be a subset of a topological space (X, τ) . If $\lambda_c d(A) \subseteq U$ for λ_c -open set U , then $\lambda_c d(\lambda_c d(A)) \subseteq U$.

Proof. Suppose $x \in \lambda_c d(\lambda_c d(A))$ but $x \notin U$. Then $x \notin \lambda_c d(A)$ and so, for some λ_c -open set V , $x \in V$ and $A \cap V \subseteq \{x\}$, but $x \in \lambda_c d(\lambda_c d(A))$ implies

$y \in \lambda_c d(A) \cap V \setminus \{x\}$ for some y . Now, $y \in U \cap V$ and $y \in \lambda_c d(A)$ and so $\emptyset \neq A \cap U \cap V \cap X \setminus \{y\} \subseteq A \cap V \subseteq \{x\}$. It follows that $x \in U$, a contradiction.

Theorem 4.17

If λ is λ -regular s-operation. Then the λ_c -derived set is g - λ_c -closed.

Proof. If A is any subset of a topological space (X, τ) with $\lambda_c d(A) \subseteq U$ for U is λ_c -open. Then by Lemma 4.16, $\lambda_c Cl(\lambda_c d(A)) = \lambda_c d(\lambda_c d(A)) \cup \lambda_c d(A) \subseteq U$.

Theorem 4.18

A subset A of a topological space (X, τ) is g - λ_c -open if and only if $F \subseteq \lambda_c Int(A)$ whenever $F \subseteq A$ and F is λ_c -closed in (X, τ) .

Proof. Let A be g - λ_c -open and $F \subseteq A$ where F is λ_c -closed. Since $X \setminus A$ is g - λ_c -closed and $X \setminus F$ is a λ_c -open set containing $X \setminus A$ implies $\lambda_c Cl(X \setminus A) \subseteq X \setminus F$. By Proposition 3.12, $X \setminus \lambda_c Int(A) \subseteq X \setminus F$. That is $F \subseteq \lambda_c Int(A)$.

Conversely, suppose that F is λ_c -closed and $F \subseteq A$, implies that $F \subseteq \lambda_c Int(A)$. Let $X \setminus A \subseteq U$, where U is λ_c -open. Then $X \setminus U \subseteq A$, where $X \setminus U$ is λ_c -closed. By hypothesis $X \setminus U \subseteq \lambda_c Int(A)$. That is $X \setminus \lambda_c Int(A) \subseteq U$ and then by Proposition 3.12, $\lambda_c Cl(X \setminus A) \subseteq U$. This implies $(X \setminus A)$ is g - λ_c -closed and A is g - λ_c -open.

Note 4.19

The union of two $g-\lambda_c$ -open sets need not be $g-\lambda_c$ -open. As it is shown in the following example:

Example 4.20

Let $X = \{a, b, c\}$, and $\tau = P(X)$. We define an s-operation $\lambda : SO(X) \rightarrow P(X)$ as:

$$\lambda(A) = \begin{cases} A & \text{if } A = \{b\} \text{ or } \phi \\ X & \text{if } A \neq \{b\} \end{cases}.$$

If $A = \{a\}$ and $B = \{c\}$, then A and B are $g-\lambda_c$ -open sets in X , but $A \cup B = \{a, c\}$ is not a $g-\lambda_c$ -open set in X .

Theorem 4.21

Let $\lambda : SO(X) \rightarrow P(X)$ be a λ -regular s-operation, and let A and B be two $g-\lambda_c$ -open sets in a space X . Then $A \cap B$ is also $g-\lambda_c$ -open.

Proof. If A and B are $g-\lambda_c$ -open sets in a space X . Then $X \setminus A$ and $X \setminus B$ are $g-\lambda_c$ -closed sets in a space X . By Theorem 4.8, $(X \setminus A) \cup (X \setminus B)$ is also $g-\lambda_c$ -closed set in X . That is $(X \setminus A) \cup (X \setminus B) = X \setminus (A \cap B)$ is a $g-\lambda_c$ -closed set in X . Therefore $A \cap B$ is a $g-\lambda_c$ -open set in X .

Theorem 4.22

A set A is $g-\lambda_c$ -open if and only if $\lambda_c \text{Int}(A) \cup X \setminus A \subseteq G$ and G is λ_c -open implies $G = X$.

Proof. Suppose that A is $g-\lambda_c$ -open in X . Let G be λ_c -open and $\lambda_c \text{Int}(A) \cup (X \setminus A) \subseteq G$. Then

$$X \setminus G \subseteq X \setminus (\lambda_c \text{Int}(A) \cup (X \setminus A)) = (X \setminus \lambda_c \text{Int}(A)) \cap A. \quad \text{That is}$$

$$X \setminus G \subseteq (X \setminus \lambda_c \text{Int}(A)) \setminus (X \setminus A). \quad \text{Thus } X \setminus G \subseteq \lambda_c \text{Cl}(X \setminus A) \setminus (X \setminus A),$$

since $X \setminus \lambda_c \text{Int}(A) = \lambda_c \text{Cl}(X \setminus A)$. Now, $X \setminus G$ is λ_c -closed and $X \setminus A$ is g - λ_c -closed, by Theorem 4.15, it follows that $X \setminus G = \emptyset$. Hence $G = X$.

Conversely, let $\lambda_c \text{Int}(A) \cup X \setminus A \subseteq G$ and G is λ_c -open, this implies that $G = X$. Let U be a λ_c -open set such that $X \setminus A \subseteq U$. Now $\lambda_c \text{Int}(A) \cup (X \setminus A) \subseteq \lambda_c \text{Int}(A) \cup U$ which is clearly, λ_c -open and so by the given condition $\lambda_c \text{Int}(A) \cup U = X$, then $(X \setminus \lambda_c \text{Int}(A)) \cap (X \setminus U) = \emptyset$, so $(X \setminus \lambda_c \text{Int}(A)) \subseteq U$ which implies that $\lambda_c \text{Cl}(X \setminus A) \subseteq U$ by Proposition 3.12. Hence $X \setminus A$ is g - λ_c -closed, therefore A is g - λ_c -open.

Theorem 4.23

Every singleton set in a space X is either g - λ_c -open or λ_c -closed.

Proof. Suppose that $\{x\}$ is not g - λ_c -open, then by definition $X \setminus \{x\}$ is not g - λ_c -closed. This implies that by Theorem 4.13, the set $\{x\}$ is λ_c -closed.

Theorem 4.24 2.3.24

If $\lambda_c \text{Int}(A) \subseteq B \subseteq A$ and A is g - λ_c -open, then B is g - λ_c -open.

Proof. Let $\lambda_c \text{Int}(A) \subseteq B \subseteq A$ implies that $X \setminus A \subseteq X \setminus B \subseteq X \setminus \lambda_c \text{Int}(A)$. That is, $X \setminus A \subseteq X \setminus B \subseteq \lambda_c \text{Cl}(X \setminus A)$ by Proposition 3.12. Since $X \setminus A$ is g - λ_c -closed, by Theorem 4.11, $X \setminus B$ is g - λ_c -closed and B is λ_c -open.

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